Reviewing Literature Relevant to a Systemic Early Numeracy Initiative: Bases of CMIT

Robert (Bob) Wright
Southern Cross University
<bwright@scu.edu.au>

Peter Gould
NSW Department of Education & Training
<peter.gould@det.nsw.edu.au>

A systemic initiative in early years mathematics—Count Me In Too (CMIT), is briefly described. This is followed by a review of links between the research base of CMIT and other research in early number. The review is organised into five areas: assessing students' knowledge and its progression; counting strategies; abstract composite unit; development of tens and ones knowledge; and knowledge of numerals. The review serves to locate the theoretical bases of CMIT more broadly in early number research.

Count Me In Too (CMIT) is a systemic initiative in early years mathematics which has operated in government schools in NSW since 1996 (Bobis & Gould, 1998, 1999; Stewart, Wright, & Gould, 1988). A basic goal of the initiative is to enhance teachers' understandings of young children's arithmetical strategies. After its initial trial in 13 schools in 1996, CMIT has received increased government funding each year. By the end of 1999, CMIT had been fully implemented in approximately 300 schools, and by 2003 will be available for implementation in all primary and central schools (approx. 1800). CMIT is also being implemented in other Australian school systems and in New Zealand on a large scale (500 teachers). Finally, CMIT theory and methods are now being applied systemically in NSW and elsewhere, at higher grade levels in primary and secondary schools (Year 7).

Development of CMIT involved application of a coherent program of research into young children's number learning and included the development of a learning framework in number which plays a key role in CMIT (NSW DET, 2000; Wright, Martland & Stafford, 2000). Presentation of an overview of this framework (Wright, 1998a) resulted in calls for examination of possible links between the framework and the underlying program of research on one hand, and other research. This paper has the purpose of examining such links. In this examination the learning framework in number and the underlying research will not be described in detail but will be alluded to as necessary. Descriptions of the research bases of the multiplication and division aspects of CMIT and their links to other research are available elsewhere (Mulligan, 1998; Mulligan & Mitchelmore, 1997) and will not be addressed here.

The examination of links between the research base of CMIT and other research is organised into five areas. These areas were determined by (a) identifying literature judged to be particularly relevant to CMIT, and (b) perusing the identified literature in order to identify common themes or issues. Thus these five areas provide a convenient way to organise the paper. Nevertheless, the areas are interrelated and many of the reports discussed here are relevant to more than one of the areas. The five areas are: assessing students' knowledge and its progression; counting strategies; abstract composite unit; development of tens and ones knowledge; and knowledge of numerals.

Assessing Students' Knowledge and its Progression

Young-Loveridge (1991) describes a longitudinal study of the development of early number knowledge. Sixty-eight children were assessed on five occasions — near the time of their 5th, 6th, 7th , 8th, and 9th birthdays. Of particular interest is the close similarity of some assessment tasks used by Young-Loveridge and the standard assessment tasks used in CMIT —tasks which are prominent in the research programs underlying development of CMIT.

Young-Loveridge selected six of the items she used in the assessment at five years of age, for which the children's scores correlate highly with their overall scores on each of the five occasions of assessment. These six items are, in Young-Loveridge's terms (pp. 110-112), forming sets, numeral identification, pattern recognition, rote counting, sequence forwards, and enumeration. The assessment of 5-year-olds also includes number sequence backwards (p. 12) and addition and subtraction with concrete and imaginary objects (p. 14-15), which also feature prominently in CMIT assessment. Young-Loveridge's study provides a very strong endorsement of the CMIT assessment tasks.

Three studies which focused on school entry knowledge and its progression during the first year of school are of interest because of similarities in their conclusions, and their relevance to CMIT. These studies are the UK study by Aubrey (1993), the New Zealand study by Young-Loveridge (1989), and the NSW study by Wright (1991, 1994). These studies detailed the wide range of levels of students' knowledge, and showed that current curricula were unsuited to many students, particularly the more knowledgeable students.

> The demonstration of such early competencies poses challenges to the conventional reception class curriculum which follows a sequence of sorting, matching and classifying, joining and separating sets, counting and ordering, recognising and writing numbers 0-10, where simple mathematic relationships may be demonstrated through the use of concrete material. ... [R]eception-age children clearly enter school having acquired already much of this mathematical content. (Aubrey, 1993, p. 39)

Denvir and Brown (1986a; 1986b) describe a hierarchical framework of 47 early numeracy skills. In this framework one skill is prerequisite to another if there is a logical reason why it depends on the other and all or nearly all children who succeeded on the harder task also succeeded on the easier one. The framework was used to chart the performance of seven pupils interviewed approximately six monthly over a period of two years. Denvir and Brown also describe two teaching studies and the use of the framework to map each participant's performance by showing skills acquired at pre-test, skills acquired at post-test, skills acquired at a delayed post-test, and skills taught (i.e., between pre- & post-test). In the case of one child the authors concluded that

> the acquired skills, whilst consistent with the hierarchical framework were not the ones that had been taught ... [and] whilst a child's performance ... will suggest which skills are likely to be acquired, it cannot be predicted precisely which skills a child will learn. (p. 152)

This finding, and Young-Loveridge's (1989) finding that "certain skills were learned over the first year of school by considerable numbers of children even though they had not been taught these skills by their teachers" (p. 56), highlight the separateness of what is taught and what children learn. As well, all of the above studies lend support to the instructional approach in CMIT. This approach highlights the need for teachers to take full account of students' current strategies and knowledge levels, and emphasises the role of problem solving and reflection in the development of more sophisticated strategies and in the extension of knowledge. As well, the approach can be contrasted with a narrow, lock-step approach emphasising learning of specific skills.

Counting Strategies

Carpenter, Fenema, Franke, Levi and Empson (1999) present an overview of the Cognitively Guided Instruction approach (CGI) to early number (see also Carpenter, Fenema & Franke, 1996; Fenema et al., 1996). CGI includes a model of a progression of strategies children use to solve addition and subtraction word problems—direct modelling, counting and derived facts. In this model "counting" refers to the relatively sophisticated strategies of counting on and back. Strategies involving other forms of counting; for example counting perceptually available items, may be used at the level of "direct modelling". Many of the strategies and observations about strategies in CGI feature in CMIT literature and its research base. The observation for example, that "[C]ounting strategies represent more than efficient procedures for calculating answers to addition and subtraction problems. They indicate a level of understanding of number concepts and an ability to reflect on numbers as abstract entities" (Carpenter et al., 1999, p. 28) is entirely consistent with CMIT's third level of early arithmetical strategies of "Counting On and Back" (also referred to as "Advanced count-by-one strategies"), and corresponds to what Steffe and colleagues called counters of abstract unit items (Steffe, von Glasersfeld, Richards, & Cobb, 1983, pp. 66-72), the sequential integration operation (Steffe & Cobb, 1988, p. 150), and the initial number sequence (Steffe, 1992, pp. 92-95):

> The deeper question ... about how the number words come to stand for summations of units, is really the heart .... The semantic links of the number words change radically as the child develops more sophisticated unit types and associated conceptual structures. The semantic link between a number word and the concept of summation of units can only be made on the level of abstract units, and it is only then that a number word can stand for such a summation of units. (Steffe et al., 1983, p. 27)

In similar vein, Carpenter et al. observed that,

> although children frequently use fingers with Counting strategies, the use of fingers does not distinguish Counting strategies from Direct Modelling strategies. ... Fingers may be used to directly model a problem or to keep track of the steps in a counting sequence. (1999, pp. 23-4)

This is entirely consistent with descriptions in CMIT which differentiate use of fingers at the Perceptual (Level 1), Figurative (Level 2) and Counting On and Back (Level 3) levels. Each is associated with profoundly different ways children think about numbers. In describing figurative counting (CMIT Level 2) Steffe (1992) wrote:

> This is quite distinct from a child simultaneously putting up four fingers and then three fingers as replacements (emphasis in original) for the hidden items, and then taking the fingers as a collection of perceptual unit items for counting. This latter coordination of finger patterns and the counting scheme is within the province of children with perceptual counting schemes. (p. 89)

As described above in the area of children's counting strategies, links between the CGI literature and the research base of CMIT can be discerned. Fuson's research on counting and related areas of early number is closely linked to the research base of CMIT.

> Children in the United States display a progression of successively more complex, abstract, efficient, and general conceptual structures for addition and subtraction. Each successive level demonstrates cognitive advances and requires new conceptual understandings. ... The work [i.e., as presented in a book chapter] on counting and cardinal conceptual units, ... cardinal conceptual operations, ... and cardinal conceptual structures ... was stimulated by Steffe and is a summary of his work and my own related work. (Fuson, 1992, p. 250)

The reports by Gray (1991) and Gray & Tall (1994) are particularly relevant to CMIT and its extension to grade levels above K-2. These focus on the strategies used by children to solve addition and subtraction tasks. The tasks were classified as Stage 1—addition and subtraction facts to 10, and Stage 2—addition and subtraction facts in the range 10 to 20. Seventy-two participants were selected from two schools considered to be typical English schools, as follows; six from each school in each of the six age ranges of 7+, 8+, ... 12+. Each of the 12 sets of six students consisted of two children at each of three teacher-defined ability levels—below average, average and above average. Participants' addition strategies were classified as one of: known fact, derived fact, count-on, or count-all. Participants' subtraction strategies were classified as one of: known fact; derived fact; count-up or count-back; or take-away. Count-up and count-back are considered analogous to count-on. The take-away strategy is considered analogous to count-all and involves three forward counts from one (e.g., counting out a known minued, then counting out a known subtrahend from the minued, then counting the remainder to obtain the unknown difference).

Gray describes these four-level classifications of addition and subtraction strategies as preferential hierarchies. If unable to solve a task by immediate recall (of a known fact) the child reverts back to what might be regarded as a preferred level. Gray identifies "two distinct approaches to the regression" (pp. 569-570):

> The first makes use of other known knowledge, the deductive approach [used mostly by above average and average children]. The second is dominated by the use of counting, the procedural approach [used mostly by below average].
>
> What has become fairly clear ... is that the below average ability child is neither successful at learning the number bonds nor in making use of the ones that they do know ... .[For younger below average children] memory is abandoned for a procedure that involves the use of physical or quasi-physical objects. The bits they do know do not appear to be held together, with the result that this change in strategy may involve the child in long sequences of counting. ... However, by the end of the middle years of schooling such children feel secure, often confident, in their procedure.
>
> In contrast, condensing the long sequences appears to be almost intuitive to the above average child. This eventually becomes the cornerstone to their higher level of attainment; they can take short cuts and operate with increasing levels of abstraction.

Gray's research underlies the importance of children learning to count-on (Level 3 of CMIT's early arithmetical strategies) and also the importance of children progressing to levels where they routinely use "non-count-by-ones" strategies, can instantly recall facts, and also use these to derive other facts (i.e., Level 4).

Thompson (1995) describes a study of the solution methods of 59 Year 2 children (6- & 7-year-olds) on simple addition, subtraction and multiplication problems. Findings suggest that as children progress through school they continue to use counting as an important part of their problem solving repertoire, combining these counting skills in idiosyncratic ways, other learned skills and acquired knowledge. Thompson argues that the importance of counting as a basic building block of numerical understanding suggests that teachers of young children may need to place greater emphasis on the development and integration of counting skills during the first few years of school. Thompson's findings and recommendations are generally consistent with the emphasis in CMIT on developing increasingly sophisticated counting and other strategies in early number learning.

Abstract Composite Unit

Steffe's (Steffe & Cobb, 1988) psychological construct of "abstract composite unit" (p. 334), and related constructs such as "numerical composite" (p. 335) and "iterable unit" (p. 335) underlie CMIT's models of the development of children's knowledge of addition and subtraction, tens and ones, and multiplication and division. These constructs are prominent in research involving analysis of children's thinking in measurement (Clements, Battista, Sarama, Swaminathan, & McMillen, 1997) and geometric (Battista & Clements, 1996; Battista, Clements, Arnoff, Battista, & Borrow, 1998; Wheatley & Reynolds, 1996;) contexts, as well as in number contexts (e.g., Hunting & Davis, 1991; Jones et al., 1996; Lamon, 1994; 1996; Saenz-Ludlow, 1994; Watanabi, 1995).

According to Lamon (1996), "the ability to form and operate with increasingly complex unit structures appears to be an important mechanism by which more sophisticated reasoning develops" (p. 170). Lamon uses the notion of composite unit in studying children's partitioning strategies (1996), and the development of proportional reasoning (1994). A report by Jones et al., (1996) for example, presents and applies a framework of the development of children's numerical thinking spanning the period from pre-place value and initial counting to the understanding of three-digit numbers. The framework consists of five levels across four key constructs of counting, partitioning, grouping and number relationships, and draws on Steffe's notions of numerical composites and abstract composite unit (p. 311). Watanabe (1995) described relationships between children's fraction concepts and whole number concepts and presented case studies based on three children's solutions of two tasks which stimulate children to think in terms of fractions. The study identifies four types of unit coordination schemes (i.e., strategies) used by the children, viz one-as-one, many-as-one, one-as-many, and many-as-many.

Wheatley and Reynolds (1996; see also Reynolds & Wheatley, 1996) used the construct of abstract composite unit to explain children's geometric thinking as well their number thinking. They describe

> a relationship between a child's ability to construct abstract units from nonrectangular shapes and their use of ten as an abstract unit in adding and subtracting whole numbers. ... We conjecture that constructing abstract units is quite a general and significant mathematical operation which transcends number. (pp. 67-68)

Empirical evidence in support of this conjecture is drawn from observations of four children over a three year period. The study argues that "classroom activities which encourage the construction of units are likely to be useful to children in coming to act mathematically" (p. 82).

Finally, Battista, Clements and colleagues used the construct of abstract composite unit and related constructs in describing students' understanding of 3D arrays of cubes (Battista & Clements, 1996), development of length concepts (Clements et al., 1997) and spatial structuring of 2D arrays of squares (Battista et al., 1998).

Development of Tens and Ones Knowledge

The model of the development of children's knowledge of tens and ones used in CMIT is based on research by Steffe and Cobb (1988) and a study by Cobb and Wheatley (1988) which draws on the research by Steffe and Cobb (Cobb & Wheatley, 1988, p. 1). The model involves "three increasingly sophisticated concepts of ten ... numerical composite, abstract composite unit, and iterable unit" (Cobb & Wheatley, 1988, p. 4). Descriptions of children's conceptual structures for multidigit numbers—unitary multidigit, sequence-tens and ones, and integrated sequence-separate tens (Fuson et al., 1997) correspond to concepts of ten described by Cobb and Wheatley (1988) and related constructs (Steffe & Cobb, 1988) as follows: unitary multidigit corresponds to numerical composite and the initial number sequence; sequence-tens and ones corresponds to abstract composite unit and the explicitly-nested number sequence; and integrated sequence-separate tens corresponds to iterable unit. As well, there is accordance between Cobb and Wheatley's (1988) analysis of concepts of ten and Kamii's (1986) levels of children's counting by tens. Related to these correspondences is the concurrence among several researchers in descriptions of two distinct types of strategies children use in 2-digit addition and subtraction. Thus Beishuizen's (1993, p. 295) two categories of Dutch second-graders' strategies, that is, 1010 (the split method) and N10 (the jump method) correspond respectively with (a) collections-based and counting-based interpretations of 2-digit numerals (Cobb & Wheatley, 1988), (b) the partial sums and cumulative sums categories of additive strategies (Thompson, 1994, p. 333), and (c) the combining tens and ones and incrementing types of invented algorithms for multidigit addition and subtraction (Carpenter et al., 1999, pp. 70-73).

Knowledge of Numerals

From the perspective of CMIT, students' developing knowledge of numerals is considered to be an important aspect of early number development and to deserve a distinctive research focus. Nevertheless, research literature on knowledge of numerals in early number seems to be rare. Wright (1998b) for example, describes observations relating to young children's knowledge of numerals and its relation to number word knowledge. In CMIT, knowledge of numerals includes: identifying (reading or naming); recognising (selecting a named numeral); writing; sequencing (e.g., putting numerals from 26-35 in correct sequence); and ordering (e.g., putting 10, 28, 21, 30 in ascending order). A view held in CMIT is that it is appropriate to teach knowledge of numerals directly, and that learning about numerals can significantly precede learning place value. Thus young children should be taught to read 2- and 3-digit numerals for example, prior to formal teaching of place value, and this can constitute an important basis for learning place value. This accords with reports by Wigley (1997) and Hewitt and Brown (1998) of systematic approaches to teaching children to name numerals as an important basis of early numeracy.

Conclusion

This paper has reviewed research literature in early number learning judged to be particularly significant to the CMIT initiative. Thus the paper serves the purpose of locating the theoretical bases of CMIT more broadly in the relatively extensive body of research in early number learning that has been conducted in the last 15-20 years, both in Australia and New Zealand, and elsewhere.

References

Aubrey, C. (1993). An investigation of the mathematical knowledge and competencies which young children bring into school. *British Educational Research Journal, 19*(1), 27-41.

Battista, M. T. & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education, 27*, 258-292.

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education, 29*, 209-532.

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education, 24*, 294-323.

Bobis J., & Gould, P. (1998). The impact of an early number project on the professional development of teachers. In C. Kanes, M. Goos & E. Warren (Eds.), *Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia (Vol. 2*, pp. 107-113*)*. Brisbane: MERGA.

Bobis, J., & Gould, P. (1999). The mathematical achievement of children in the Count Me In Too Program. In Truran, J. & Truran, K., *Making the difference*. Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australasia (pp. 84-90). Sydney: MERGA.

Carpenter, T. P., Fennema, E., Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics. *The Elementary School Journal, 97*(1), 3-20.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Clements, D. H., Battista, M. T., Sarama, J., Swaminathan, S., & McMillen, S. (1997). Students' develop-ment of length concepts in a Logo-based unit on geometric paths. *Journal for Research in Mathematics Education, 28*, 70-95.

Cobb, P., & Wheatley, G. (1988). Children's initial understandings of ten. *Focus on Learning Problems in Mathematics, 10*(3), 1-26.

Denvir, B. & Brown, M. (1986a). Understanding of number concepts in low attaining 7-9 year olds: Part 1. Development of descriptive framework and diagnostic instrument. *Educational Studies in Mathematics, 17*, 15-36.

Denvir, B. & Brown, M. (1986b). Understanding of number concepts in low attaining 7-9 year olds: Part II. The teaching studies. *Educational Studies in Mathematics, 17*, 143-164.

Fuson, K. C. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching & learning* (pp. 243-275). New York: MacMillan.

Fuson, K. C., Wearne, D., Hiebert, J., Human, P., Olivier, A., Carpenter, T., & Fenema, E. (1997). Children's conceptual structure for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28,* 130-162.

Fenema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V, & Empson, S. (1996). Learning to use children's thinking in mathematics instruction: A longitudinal study. *Journal for Research in Mathematics Education, 27,* 403-434.

Gray, E. M. (1991). An analysis of diverging approaches to simple arithmetic: Preference and its consequences. *Educational Studies in Mathematics, 22,* 551-74.

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education, 25,* 116-140.

Hewitt, D. & Brown, E. (1998). On teaching early number through language. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd conference of the International Group for the Psychology of Mathematics Education, (Vol. 3,* 41-48*).* Stellenbosh, South Africa: University of Stellenbosh.

Hunting, R. P., & Davis, G. (1991). *Early fraction learning.* New York: Springer-Verlag.

Jones, G. A., Thornton, C. A., Putt, I. J., Hill, K. M., Mogill, A. T., Rich, B. S., Van Zoest, L. R. (1996). Multidigit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education, 27,* 310-336.

Kamii, C. (1986). Place value: An explanation of its difficulty and educational implications for the primary grades. *Journal of Research in Early Childhood Education, 1,* 75-86.

Lamon, S. J. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 89-121*).* Albany, NY: SUNY Press.

Lamon, S. J. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education, 27,* 170-193.

Mulligan, J. T. (1998). A research-based framework for assessing early multiplication and division. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times.* Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, (Vol. 2, pp. 404-411). Brisbane: MERGA.

Mulligan, J. T. & Mitchelmore, M.. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28,* 309-330.

NSW Department of Education and Training. (2000*). Count Me In Too: Professional Development Package.* Sydney: Author.

Reynolds, A. & Wheatley, G. H. (1996). Elementary students' construction and coordination of units in an area setting. *Journal for Research in Mathematics Education, 27,* 564-581.

Saenz-Ludlow, A. (1994). Michael's fraction schemes. *Journal for Research in Mathematics Education, 25,* 50-85.

Steffe, L. P. (1992). Learning stages in the construction of the number sequence. In J. Bideaud, C. Meljac, & J. Fischer (Eds.), *Pathways to number: Children's developing numerical abilities, (pp. 83-98).* Hillsdale, NJ: Lawrence Erlbaum.

Steffe, L. P., & Cobb, P. (with E. von Glasersfeld). (1988). *Construction of arithmetic meanings and strategies.* New York: Springer-Verlag.

Steffe, L. P., von Glasersfeld, E., Richards, J. & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application.* New York: Praeger.

Stewart, R., Wright, R. J., & Gould, P. (1998). Kindergartener's progress in the Count Me In Too Project. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times* Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia, (Vol. 2, 556-563). Brisbane: MERGA.

Thompson, I. (1994). Young children's idiosyncratic written algorithms for addition. *Educational Studies in Mathematics, 26,* 323-45.

Thompson, I. (1995). The role of counting in the idiosyncratic mental calculation algorithms of young children. *European Early Childhood Education Research Journal, 3*(1), 5-16.

Watanabe, T. (1995). Coordinations of units and understanding of simple fractions: Case Studies. *Mathematics Education Research Journal, 7,* 160-175.

Wheatley, G. H. & Reynolds, A. (1996). The construction of abstract units in geometric and numerical settings. *Educational Studies in Mathematics, 30,* 67-83.

Wigley, A. (1997). Approaching number through language. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 113-22*).* Buckingham, UK: Open University Press.

Wright, R. J. (1991). What number knowledge is possessed by children entering the kindergarten year of school? *The Mathematics Education Research Journal, 3*(1), 1-16.

Wright, R. J. (1994). A study of the numerical development of 5-year-olds and 6-year-olds. *Educational Studies in Mathematics, 26, 25-44.*

Wright, R. J. (1998a). An overview of a research-based framework for assessing and teaching early number. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times.* Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia, (Vol. 2, 701-708*).* Brisbane: MERGA.

Wright, R. J. (1998b).Children's beginning knowledge of numerals and its relationship to their knowledge of number words: An exploratory, observational study. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd conference of the International Group for the Psychology of Mathematics Education, Vol. 4*, 201-208. Stellenbosh, South Africa: University of Stellenbosh.

Wright, R. J., Martland, J. R., & Stafford, A. (2000*). Early numeracy: Assessment for teaching and intervention.* London: Sage / Paul Chapman Publications.

Young-Loveridge, J. (1989). The development of children's number concepts: The first year of school. *New Zealand Journal of Educational Studies, 24*(1*),* 47-64.

Young-Loveridge, J. M. (1991). The development of children's number concepts from ages five to nine, Volumes 1 & 2. Hamilton, NZ: University of Waikato.